Math 242

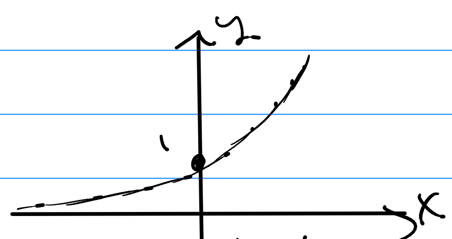
Note:
- Webassign is all due.
- Know ch6 examples for final.

Final Review: $\boxed{1^{30} - 2^{30} \quad 336\ JB}$

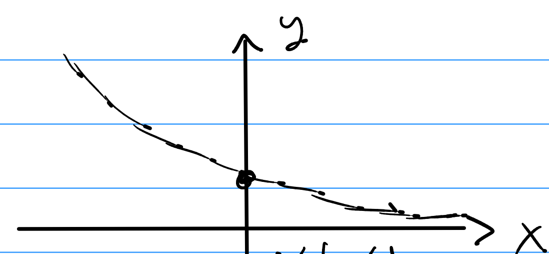
Ch6

	b^x	$\log_b x$
Domain:	$(-\infty, \infty)$	$(0, +\infty)$
Range:	$(0, +\infty)$	$(-\infty, +\infty)$

b^x

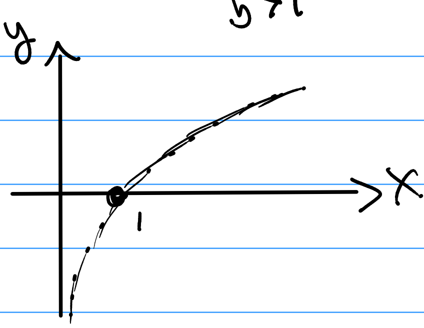


$b > 1$

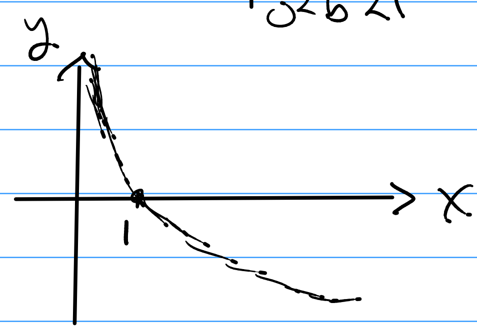


$0 < b < 1$

$\log_b x$



$b > 1$



$0 < b < 1$

Calculus $D_x[e^x] = e^x$

1. $\int e^x\,dx = e^x + C$

$D_x[\ln(x)] = \frac{1}{x}$

(2) $\int \frac{1}{x}\,dx = \ln|x| + C$

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any b $\boxed{\log_b x = \frac{1}{\ln b} \ln x}$

b/c $y = \log_b x$ iff $b^y = x$

$\ln(b^y) = \ln(x)$

$y \ln(b) = \ln(x)$

$y = \frac{\ln(x)}{\ln(b)}$

$\boxed{b^x = e^{x(\ln b)}}$

(1) $D_x[b^x] = \ln(b)\, b^x \rightarrow \int b^x\,dx = \frac{1}{\ln b} b^x + C$

(2) $D_x[\log_b x] = \frac{1}{\ln(b)} \frac{1}{x}$

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ex $f(t) = \sin(e^{t^2+1})$

$f'(t) = \cos(e^{t^2+1}) \frac{d}{dt}(e^{t^2+1})$

$= \cos(e^{t^2+1})\, e^{t^2+1} \frac{d}{dt}[t^2+1]$

$= \cos(e^{t^2+1})\, e^{t^2+1} (2t)$

$$\boxed{f'(t) = 2t\, e^{t^2+1} \cos\left(e^{t^2+1}\right)}$$

ex $\displaystyle\int \tan(\theta)\, d\theta$ vs $\displaystyle\int (\sec^2\theta)\, d\theta$

$= \tan\theta + C$

b/c $D_\theta[\tan\theta] = \sec^2\theta$

$$= \int \frac{\sin\theta}{\cos\theta}\, d\theta$$

$u = \cos\theta$

$du = -\sin\theta\, d\theta$

$$= -\int \frac{1}{u}\, du = -\ln|u| + C$$

$$= \boxed{-\ln|\cos\theta| + C}$$

$$= \ln|\cos\theta|^{-1} + C$$

$$= \boxed{\ln|\sec\theta| + C}$$

$$\boxed{\int \tan\theta\, d\theta = \ln|\sec\theta| + C}$$

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ex $f(x) = \dfrac{x}{1 - \ln(x-1)}$

Domain: ① $x - 1 > 0 \to x > 1$

and $1 - \ln(x-1) \neq 0$

Find $1 - \ln(x-1) = 0$

$\log_e(x-1) = 1 \to (x-1) = e^1$

$x = e + 1$

graph

$f(x) = \dfrac{x}{1-\ln(x-1)}$ **Domain** $x>1,\ x\neq e+1$

$(1, e+1) \cup (e+1, +\infty)$

Intercepts ① no y-intercept (b/c $x>1$)

② x-intercept $\nexists$ no $y=0$ $0 = \dfrac{x}{1-\ln(x-1)}$

$\to$ ($x=0$) not in Domain.

asymptotes:

Vertical zeros of denominator $\dfrac{x}{1-\ln(x-1)}$

$x = e+1$

① $\displaystyle\lim_{x\to(e+1)^+} \frac{x}{1-\ln(x-1)} = \lim_{x\to(e+1)^+} \frac{e+1}{1-\ln(x-1)} = -\infty$

② $\displaystyle\lim_{x\to(e+1)^-} \frac{x}{1-\ln(x-1)} = +\infty$

Horz $\displaystyle\lim_{x\to+\infty} \frac{x}{1-\ln(x-1)} = ?$

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f' $f(x) = \dfrac{x}{1-\ln(x-1)}$

$$f'(x) = \frac{(1)(1-\ln(x-1)) - (x)\left(-\frac{1}{x-1}\cdot 1\right)}{(1-\ln(x-1))^2}$$

$$f'(x) = \frac{(1-\ln(x-1))(x-1)+x}{(1-\ln(x-1))^2(x-1)}$$

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ex) $$\int \frac{\cos(\ln x)}{x}\,dx = \int \cos u\,du$$

$u = \ln x$

$du = \frac{1}{x}dx$

$$= \sin u + C$$

$$= \boxed{\sin(\ln(x)) + C}$$

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ex) $$\int \frac{\sin 2x}{1+\cos^2 x}\,dx$$

Know: $\sin 2x = 2\sin x\cos x$

let $u = 1+\cos^2 x$

$du = -\boxed{2\cos x \sin x\,dx}$

$\sin 2x\,dx$

$$\to -\int \frac{1}{u}\,du = -\ln|u| + C$$

$$= \boxed{-\ln|1+\cos^2 x| + C}$$

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ex) $$\int \cot x\,dx = \int \frac{\cos x}{\sin x}\,dx = \int \frac{1}{u}\,du = \ln|u| + C$$

let $u = \sin x$

$du = \cos x\,dx$

$$= \boxed{\ln|\sin x| + C}$$

vs) $$\int \tan x\,dx = \boxed{-\ln|\cos x| + C} = \ln|\sec x| + C$$

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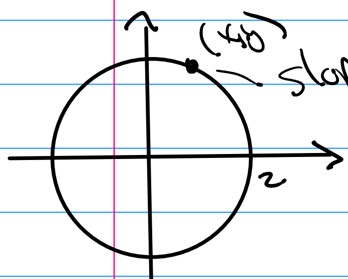
logarithmic differentiation

use $\ln(a^b) = b\ln(a)$, $\ln(ab) = \ln(a) + \ln(b)$

$\ln\left(\frac{a}{b}\right) = \ln(a) - \ln(b)$

with implicit differentiation

ex Implicit Derivatives: $\boxed{x^2 + y^2 = 4}$



$$2x + 2y \cdot y' = 0$$

$$\boxed{y' = -\frac{x}{y}}$$

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ex $y = \dfrac{e^{-x}\cos^2 x}{x^2 + x + 1}$

explicit Derivatives: $y' = D_x\left[\dfrac{e^{-x}\cos^2 x}{x^2+x+1}\right]$

→ need: quotient rule
product rule
chain rule
exponents
trig.
polynomials

vs

$$y = \frac{e^{-x}\cos^2 x}{x^2+x+1} \rightarrow \ln(y) = \ln\left[\frac{e^{-x}\cos^2 x}{x^2+x+1}\right]$$

$$\rightarrow \ln y = (-x) + 2\ln(\cos x) - \ln(x^2+x+1)$$

use Implicit Deriv.

$$\frac{1}{y}y' = -1 + \frac{-2\sin x}{\cos x} - \frac{2x+1}{x^2+x+1}$$

$$y' = y\left[-1 - 2\tan x - \frac{2x+1}{x^2+x+1}\right]$$