Math 243

Syllabus

Math = toys (+) rules

$\boxed{\text{ex}}$ numbers (rs) $+, -, *, \div, x^2$

Solve: $3x + 7 = x^3 + 2x^2 + 1 - 7$
$\quad\;\; -7$

Problem Solving

(*) 0) "what do you bring to the game?" $\boxed{\text{toolbox}}$

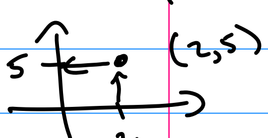
1) read the problem
2) plan
3) carry out the plan
4) check

Calculus

toys: Functions $\qquad f : \text{Domain} \to \text{Codomain}$

$\boxed{\text{ex}}$ $f(a) = b$ $\qquad a \xrightarrow{f} b$

typically in calculus $\quad f : \underbrace{\boxed{\text{Real Numbers}}}_{\mathbb{R}} \to \underbrace{\boxed{\text{Real Numbers}}}_{\mathbb{R}}$



$f(x) = x^2 + 1 \qquad f(\square) = \square^2 + 1$

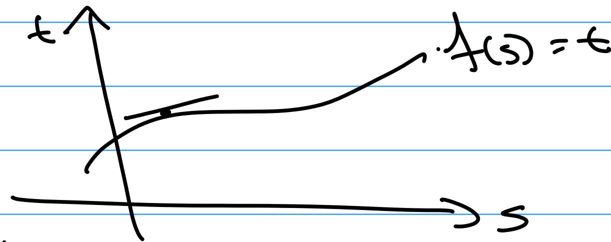
$2 \xrightarrow{f} 5$ $\quad \boxed{f(2) = 2^2 + 1 = 5}$ $\quad f(2) = 2^2 + 1$

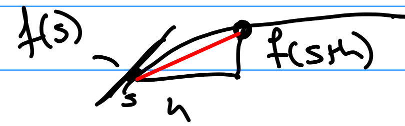
$(2,5)$ $\qquad f(x+h) = (x+h)^2 + 1$

tools: functions $\qquad$ rules: $f+g,\ f-g,\ f\cdot g,\ f/g$

$$(f\circ g)(s) = f(g(s))$$

Calculus





$$f'(s) = \lim_{h\to 0} \frac{f(s+h) - f(s)}{h}$$

Notations

$$y = f(x) \xrightarrow{\text{Derivative}} f'(x),\ \frac{d}{dx}[f(x)],\ \frac{dy}{dx},\ D_x[f(x)]$$

$$[f(x)]'$$

$$\boxed{\frac{d}{dx}[x^a]} = \lim_{h\to 0} \frac{(x+h)^a - (x)^a}{h} = \boxed{\cdots} = \boxed{a\,x^{a-1}}$$

idea: $(1+x)^{107} = 1\cdot x^0\cdot(1)^{107} + 107\,x^1(1)^{106} + \frac{107!}{2!\,105!}\,x^2(1)^{105} + \frac{107!}{3!\,104!}\,x^3 + \ldots$

$7! = 7\cdot 6\cdot 5\cdot 4\cdot 3\cdot 2\cdot 1$

$+ x^{107}$

ex $\quad \frac{d}{dx}[x^3] = 3x^2$

Derivative Toolbox

$$\frac{d}{dx}(x^a) = a\,x^{a-1}$$

$$\frac{d}{dx}[f \pm g] = f' \pm g'$$

$$\frac{d}{dx}\left[3x^2 + 7x^{1/2} + 2x^{-1/3}\right]$$

$$6x + \tfrac{7}{2}x^{-1/2} - \tfrac{2}{3}x^{-4/3}$$

$$\frac{d}{dx}[f \cdot g] = f'g + fg'$$

$$\frac{d}{dx}\left\{ f/g \right\} = \frac{f'g - fg'}{g^2}$$

$$\frac{d}{dx}\left[f(g(x)) \right] = f'(g(x)) \cdot g'(x) \quad \star$$

trig $\frac{d}{dx}[\sin(x)]$, $\frac{d}{dx}\{\cos(x)\}$, ...

ex $$\frac{d}{dx}\left[\sin(x)\cos x + \left(\frac{x^2-3}{2x+1} \right)^{1/2} \right]$$

$$(\cos x)(\cos x) + (\sin x)(-\sin x) \qquad \frac{1}{2}\left(\frac{x^2-3}{2x+1}\right)^{-1/2} \cdot \frac{d}{dx}\left\{\frac{x^2-3}{2x+1}\right\}$$

$$= \boxed{\cos^2 x - \sin^2 x + \frac{1}{2}\left(\frac{x^2-3}{2x+1}\right)^{-1/2} \frac{(2x)(2x+1) - (x^2-3)(2)}{(2x+1)^2}}$$

Note:

$$\sin(x+y) = \sin x \cos y + \cos x \sin y$$

$$\cos(x+y) = \cos x \cos y - \sin x \sin y$$

$$\cos(2x) = \cos^2 x - \sin^2 x$$

Math Books

Undo?: $\frac{d}{dx}\left[x^3+3x^{1/2}-4\right] = 3x^2+\frac{3}{2}x^{-1/2}$

ex $\frac{d}{dx}\left[x^{a}\right] = a\,x^{a-1}$

$$\int x^p\,dx = \frac{1}{p+1}x^{p+1}+C$$

$$\int \sin(\boxed{3x^3+1})\cdot\boxed{(9x^2)\,dx}$$

u-substitution. let $u = 3x^3+1$

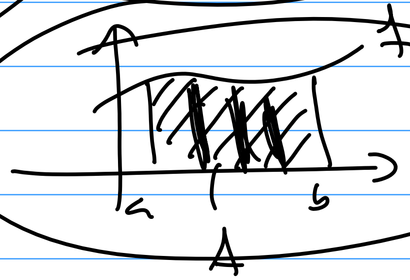
$du = 9x^2\cdot dx$

$$\int \sin(u)\,du$$

$$= -\cos u + C = -\cos(3x^3+1) + C$$

$F(x) = \int f\,dx$

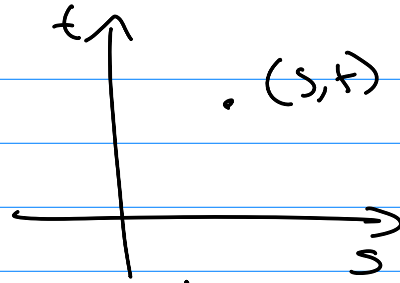
$\int_a^b f\,dx$



$A = F(b) - F(a)$

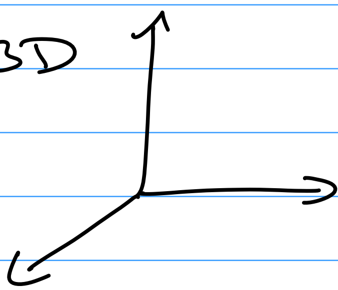
Ch 12

So far 2D-space



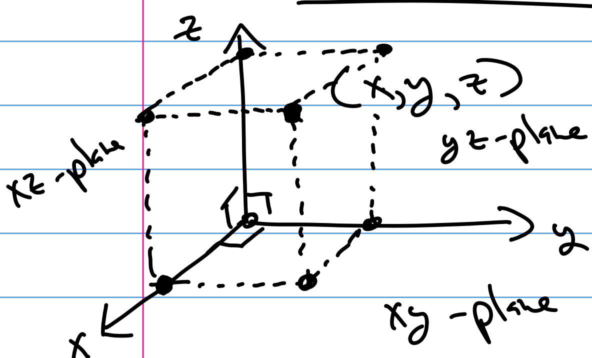
$y = f(x)$ $\qquad$ $t = f(s)$

3D



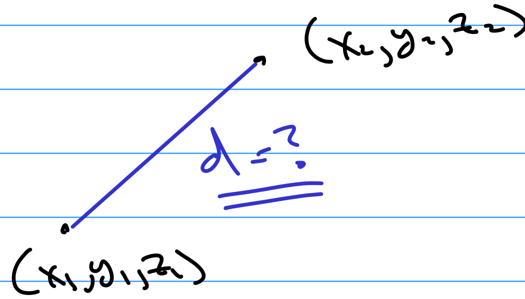
12.1 3D coordinates

$\boxed{\text{Cartesian Coord.}}$

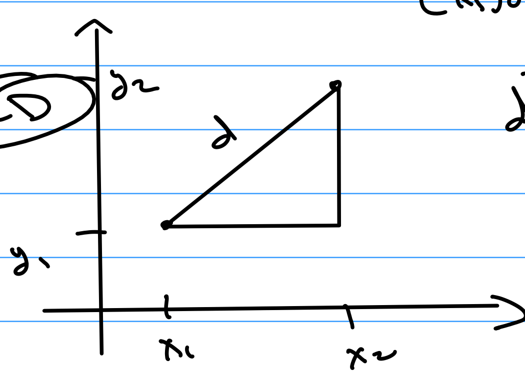


placement.

→ distance?



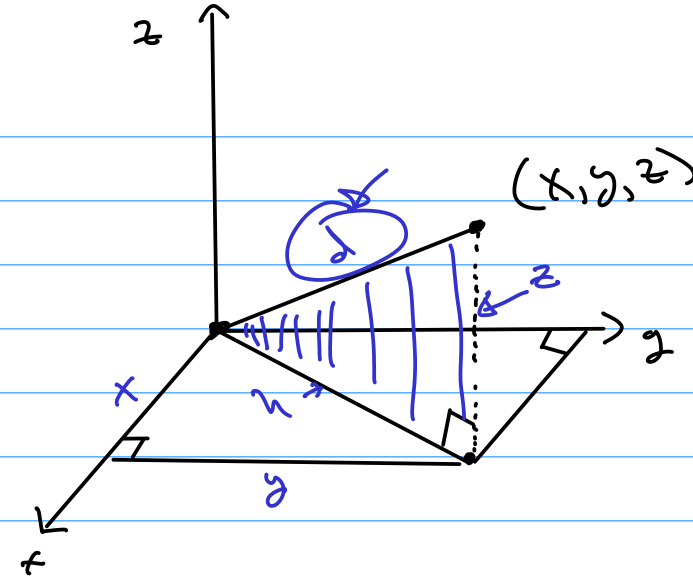
2D



$$d^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$$

$$d = \left((x_2 - x_1)^2 + (y_2 - y_1)^2 \right)^{1/2}$$

3D?



$$h^2 = x^2 + y^2$$

$$d^2 = h^2 + z^2$$

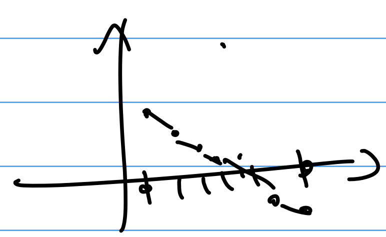
$$d^2 = x^2 + y^2 + z^2$$

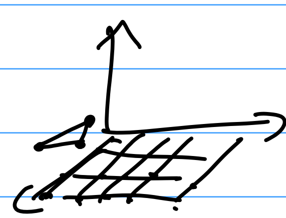
$$d = \left(x^2 + y^2 + z^2\right)^{1/2}$$

Distance Formula in 3D

$$d = \left((x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2 \right)^{1/2}$$

Objects in 2D		3D	
points	(x,y)		(x,y,z)
distance	$d = (\Delta x^2 + \Delta y^2)^{1/2}$		$d = (\Delta x^2 + \Delta y^2 + \Delta z^2)^{1/2}$
curve:	$y = f(x)$	surface:	$z = f(x,y)$
explicit:	$y = x^2 + 2x + 1$	explicit:	$z = x + y^2$
Implicite	$y^2 + x^2 = 2x$	implicit:	$x + y + z = 3$





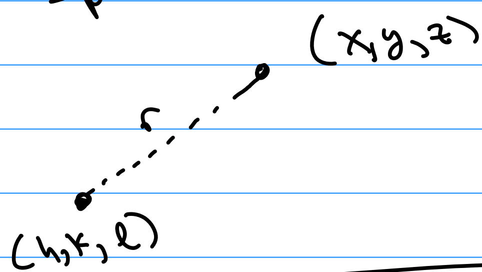
Functions to Know (the graph of)

2D: $x^2, x^3, |x|, x, \sin(x), \cos(x), \ldots$

$x^{1/2}, \frac{1}{x}, \frac{1}{x^2}, x^{1/3}, \ldots$

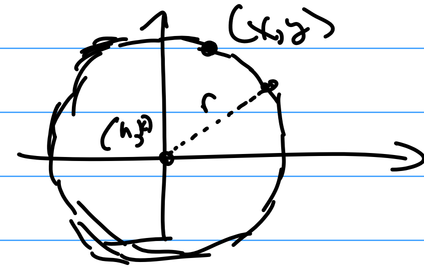
ex: $y = -(x+3)^2 - 7$

3D Sphere



circle

eqn: $(x-h)^2 + (y-k)^2 = r^2$



$$\boxed{\boxed{(x-h)^2} + (y-k)^2 + (z-\ell)^2 = r^2}$$

?

$$x^2 - 2x \; {}^{+1} + \boxed{y^2 - 4y} \; {}^{+4} + \boxed{z^2 - \tfrac{1}{4}z} \; {}^{+1/64} = 0 + 1 + 4 + \tfrac{1}{64}$$

Note: ex $(x-h)^2 = x^2 - 2xh + h^2$

$$(x-1)^2 = x^2 - 2x + 1$$

$$(x-1)^2 + (y-2)^2 + (z - \tfrac{1}{8})^2 = 5 + \tfrac{1}{64}$$