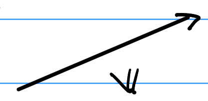
Math 243

Q's $y = \dfrac{2x}{9-\tan(x)}$ $\quad y' = \dfrac{(2x)'(9-\tan x) - (2x)(9-\tan(x))'}{(9-\tan(x))^2}$

$$y' = \frac{(2)(9-\tan x) - (2x)(0 - \sec^2(x))}{(9-\tan(x))^2} = \frac{18 - 2\tan(x) + 2x\sec^2 x}{(9-\tan x)^2}$$

Ch 12 2D, 3D, Vectors



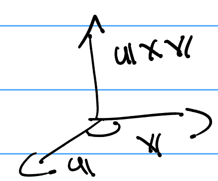
$\mathbf{v} = \langle v_1, v_2, v_3 \rangle$

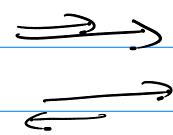
$\mathbf{v} = v_1\mathbf{i} + v_2\mathbf{j} + v_3\mathbf{k}$

$\mathbf{v} = |\mathbf{v}| \langle \cos\alpha, \cos\beta, \cos\gamma \rangle$

Ops: $+, -, c\mathbf{v}, \mathbf{u}\cdot\mathbf{v}, \mathbf{u}\times\mathbf{v}$

$$0 \le |\mathbf{u}\cdot\mathbf{v}| \le |\mathbf{u}|\,|\mathbf{v}|$$





Ca5 Application of Deriv. / Anti Derivatives

Differential Equations (Seperable Diff. Eq)

Equation: expression = expression ← Math symbols < variables / operators

Solve What makes the equality true?

ex Algebra $2x^3 - 3x + 1 = 2x - 4$

what x's (numbers)

Differential Eqn $\qquad \frac{dy}{dx} + 2x = 3y - 1$ $\qquad$ object. $\quad y = f(x)$

Solve: Find a $\boxed{y = f(x)}$ that is true!

function

$$y'' + y = 0 \to y'' = -y \quad \text{guess} \quad y = \sin x \text{ or } y = \cos x$$

Separable 1st order Diff Eq.

$$\frac{dy}{dx} = (f(y))(g(x))$$

Separate y, x

$$\frac{1}{f(y)}\,dy = g(x)\,dx$$

Integrate both sides

$$\int \frac{1}{f(y)}\,dy = \int g(x)\,dx$$

ex $\qquad \boxed{y' = \frac{x}{y}} \to \frac{dy}{dx} = \frac{x}{y} \to y\,dy = x\,dx$

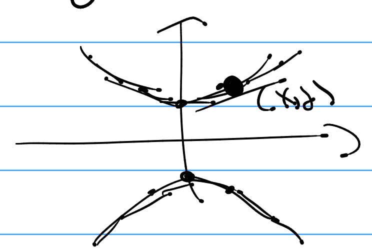
$$\to \int y\,dy = \int x\,dx$$

$$\frac{1}{2}y^2 = \frac{1}{2}x^2 + C$$

$$y^2 = x^2 + C$$

$$y^2 - x^2 = C$$

$y^2 - x^2 = 1$



Note:

$$\boxed{\begin{array}{l} y = f(x) \\ \text{ex } y = \sqrt{x^2 - 1} \end{array}} \quad \text{vs} \quad \boxed{\begin{array}{l} y, x \text{ are mixed up} \\ y^2 + x^2 = 1 \end{array}}$$

y is an explicit representation

y is an implicit representation $\boxed{y = f(x)}$

Note: Implicit Derivative

ex) $y^2 - x^2 = c$

$$\frac{d}{dx}[y^2 - x^2] = \frac{d}{dx}[c]$$

$$2y \cdot y' - 2x = 0$$

$$y' = \frac{x}{y} \quad \checkmark$$

6.5 $y' \propto y \rightarrow y' = ky$

$\propto$: proportional; k: constant of proportionality

$$\frac{dy}{dt} = ky \rightarrow \frac{1}{y} dy = k\,dt$$

$$\Rightarrow \int \frac{1}{y} dy = \int k\,dt$$

$$\ln|y| = kt + c$$

if y is only pos.

$$\boxed{\ln y = kt + c}$$

Note: Identity and Inverse Issues

→ on operators with objects.

ex Numbers and Sum

Identity $n + 0 = n$ $\quad n \cdot 1 = n$

Inverse $n + (-n) = 0$ $\quad n \cdot \frac{1}{n} = 1$

Functions: Composition

$$(f \circ g)(x) = f(g(x))$$

Identity: $(f \circ I)(x) = f(x)$

$$f(I(x)) = f(x) \rightarrow I(x) = x$$

Inverse: $(f \circ f^{-1})(x) = x$

$$f(f^{-1}(x)) = x$$

Step 1 Does $f(x)$ have an inverse? one-to-one function

$$y = f(x) \longleftrightarrow x = f(y)$$

$$f^{-1}(x) = y$$

ex $y = x^3 \longrightarrow x = y^3$

$$y = x^{1/3}$$

$$f(x) = x^3$$

$$f^{-1}(x) = x^{1/3}$$

So

if $\ln y = kt + C$

b/c $e^{\ln \square} = \square$ or $\ln e^{\square} = \square$

$$y = e^{kt+C} \rightarrow y = e^{kt} \cdot e^{C}$$

$$\rightarrow \boxed{y = \boxed{C} e^{\boxed{kt}}} \leftarrow \quad y' = ky$$

If you were given so y @ $t=0 \rightarrow y(0) = y_0$

$$\rightarrow y_0 = C e^{k \cdot 0} \rightarrow C = y_0$$

$$\boxed{y = y_0 e^{kt}} \qquad y' \propto y$$

y_0 is amount of y at $t=0$

k is const. of proportion & to find we need one point on curve

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Population: $P' = kP \rightarrow \dfrac{P'}{P} = k$] relative growth rate

Soln $\boxed{P(t) = P_0 e^{kt}}$

$\boxed{\text{ex 3}}$ $k = 0.7944$ $P_0 = 2$

$$P = 2e^{0.7944t}$$

a) pop. in 1wk?: $P(7) = 2e^{0.7944(7)} = 520.06$

$$\boxed{520}$$

b) when will pop. double?: $4 = 2e^{0.7944t}$

$$2 = e^{0.7944t} \to \ln(2) = \ln(e^{\boxed{0.7944t}})$$

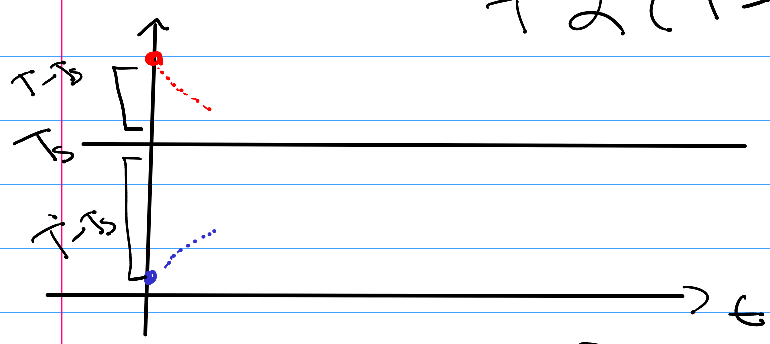
$$\ln(2) = 0.7944t \to \frac{\ln(2)}{0.7944} = t$$

Note: double: $2\cancel{P_0} = \cancel{P_0} e^{kt}$

$$2 = e^{kt} \to \boxed{\frac{\ln 2}{k} = t}$$

Newton's Law of Cooling

$$T' \propto (T - T_s) \to T' = k(T - T_s)$$



Know $\boxed{\begin{array}{l} y' = ky \\ y = y_0 e^{kt} \end{array}}$

$T' = k(T - T_s)$

let $(T - T_s) = y$

$(T - T_s)' = (y)'$

$T' = y'$

Substitute: $y' = ky$

$y = y_0 e^{kt}$

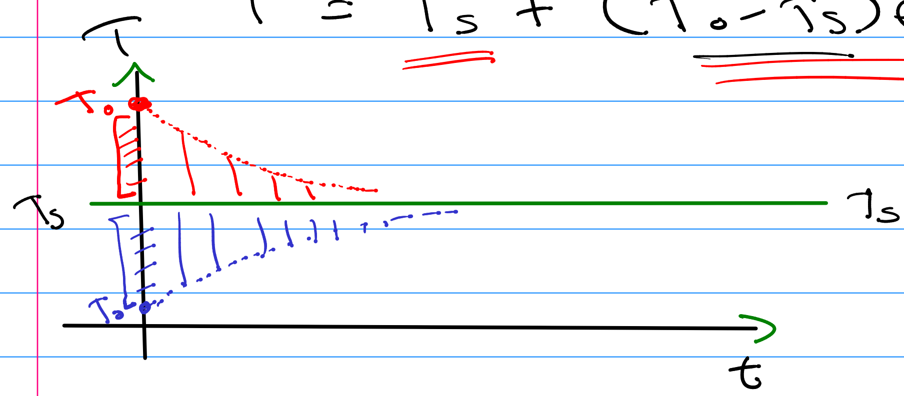
$$y = y_0 e^{kt} \quad \text{but} \quad y = [T - T_s]$$

$$T - T_s = y_0 e^{kt} \qquad y_0 = y(0)$$

?

$$y_0 = T_0 - T_s$$

$$T = \underline{T_s} + \underline{(T_0 - T_s)} e^{kt}$$



Ex: Mark is dead at $t=0$ and $T_0 = 95°F$
wait 30 mins and $T(30) = 85°F$ and $T_s = 73°F$

when did Mark die?

$$T = T_s + (T_0 - T_s) e^{kt}$$

$$T = 73 + (95 - 73) e^{kt}$$

$$T = 73 + 22 e^{kt}$$

know $t = 30\,\text{min}$, $T = 85$

$$\rightarrow \quad 85 = 73 + 22 e^{30k}$$

$$\ln(12/22) = 30k \rightarrow k = \frac{1}{30} \ln(6/11)$$

$$\boxed{T = 73 + 22 e^{\frac{1}{30}\ln(6/11)\, t}}$$

when did Mark die? $\equiv$ ($t = ?$) for $T = 98.6°F$

$$98.6 = 73 + 22 e^{1/30 \ln(6/11)\, t}$$

$$\frac{25.6}{22} = e^{\frac{1}{30}\ln(6/11)\,t}$$

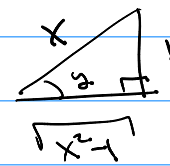
$$\ln\left(\frac{25.6}{22}\right) = \frac{1}{30}\ln(6/11)\,t$$

$$t = \frac{30\ln\left(\frac{25.6}{22}\right)}{\ln(6/11)} \approx -7.5 \approx \boxed{7\text{ min } 30\text{ sec}}$$

$y = \csc(x)$ $\boxed{\text{Inv}}$

$x = \csc(y)$

$y = \csc^{-1}(x)$



$$\frac{d}{dx}[x] = \frac{d}{dx}[\csc(y)]$$

$$1 = -\overset{x}{\csc(y)}\,\overset{\sqrt{x^2-1}}{\cot(y)}\,\frac{dy}{dx}$$

$$\frac{-1}{x\sqrt{x^2-1}} = \frac{d}{dx}\left[\csc^{-1}(x)\right]$$