

Math 451

Q's

$$Ax = b$$

Math 511

$$x = A^{-1}b$$

"Same" answer for x

linsolve(A, b)

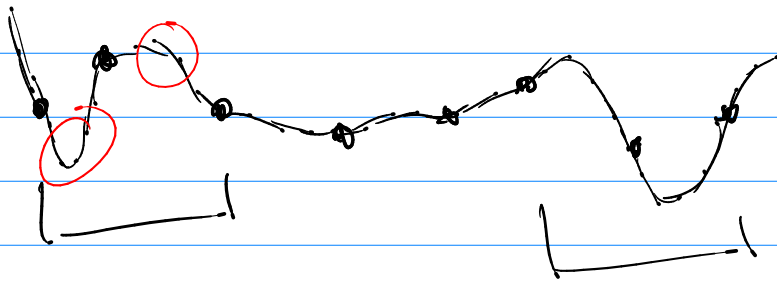
linsolve(A, b)

inv(A) * b

A \ b

Interpolating Polynomial

- ① K -data points
- ② polynomial of K terms
(degree = $K-1$)



as K grows

Data fitting

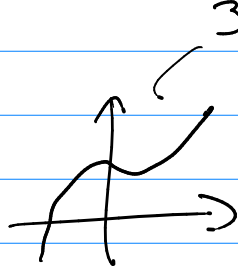
- ① given K data points
- ② "fit" the data points with a polynomial of $< K$ terms
(degree $< K-1$)

Ex

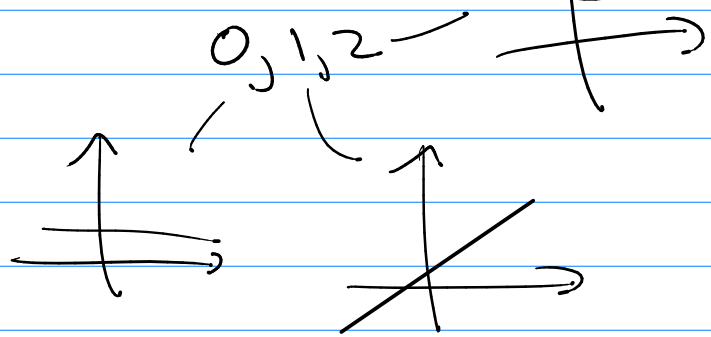
Data

4

polynomial
interpolant
degree



data fitting
degrees



20

19

0, 1, 2, ..., 19

$$y = c_1 x^n + c_2 x^{n-1} + \dots + c_{n+1} x^0 \quad \text{degree } n \text{ poly.}$$

(n+1) coeff

if data is actually represented by this poly (like it is for a polynomial interpolant) then...

system of eqns

$$y_k = c_1 (x_k)^n + c_2 (x_k)^{n-1} + \dots + c_{n+1} (x_k)^0$$

in matrix form

$$\begin{bmatrix} x_1^n & x_1^{n-1} & \dots & x_1 & 1 \\ x_2^n & x_2^{n-1} & \dots & x_2 & 1 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ x_k^n & x_k^{n-1} & \dots & x_k & 1 \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \\ \vdots \\ c_{n+1} \end{bmatrix} = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_k \end{bmatrix}$$

$$C = Y$$

except

$$Vc = y$$

interpolating polynomial

$$\begin{matrix} V & c & = & y \\ K \times K & K \times 1 & & K \times 1 \end{matrix}$$

Data fitting

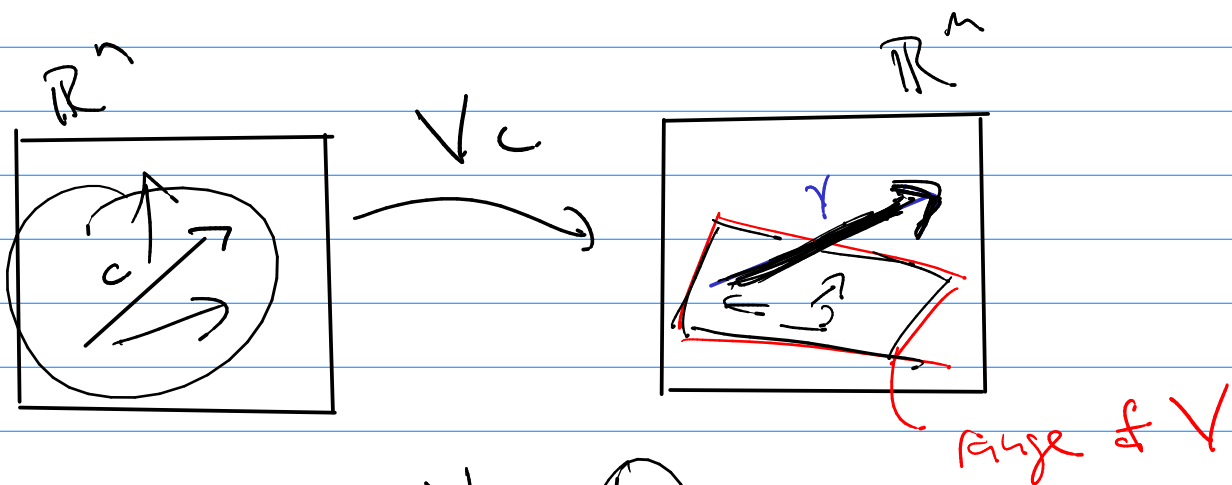
Overdetermined system

$$\begin{matrix} V & c & = & y \\ K \times n & n \times 1 & & K \times 1 \\ n+1 < K \end{matrix}$$

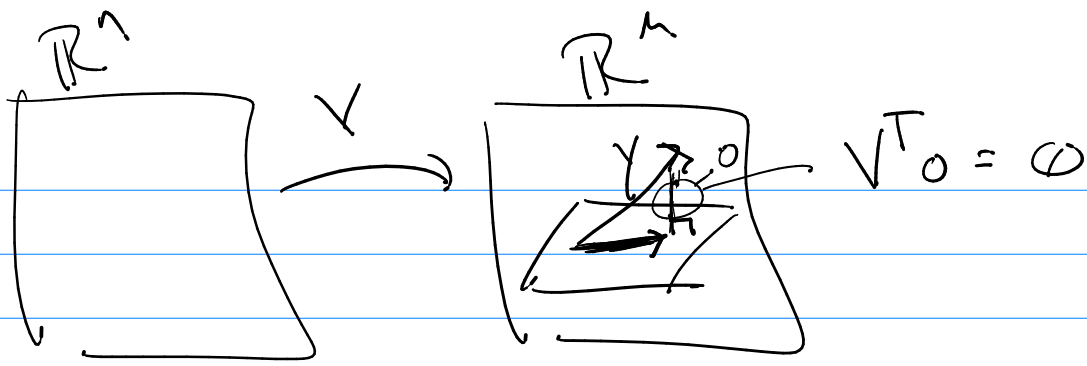
typically has no soln.

$$Vc = y$$

V is $n \times n$ C is $n \times 1$ y is $n \times 1$
 $m > n \rightarrow$ overdetermined



So solve $Vc = y$



So $Vc = y$ (no-soln b/c over det)

Solve: $V^T Vc = V^T y$ is now a determined system.

Size:

$$\begin{matrix} \begin{matrix} V^T & V \\ n \times n & m \times n \end{matrix} & c & = & \begin{matrix} V^T & y \\ n \times n & m \times 1 \end{matrix} \\ n \times n & & & n \times 1 \end{matrix}$$

$\rightarrow c$ is the least squares soln to $Vc = y$
(over det)