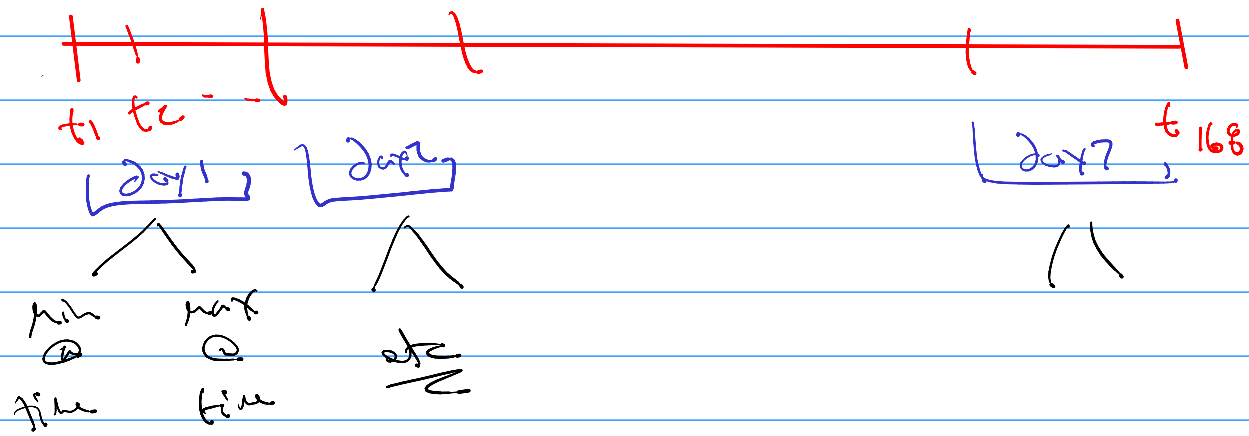
Math 451

~~Q's~~ 7 days → $7\cdot(20+4) = 168$

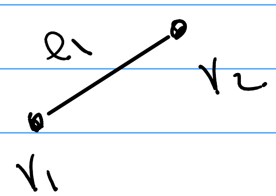


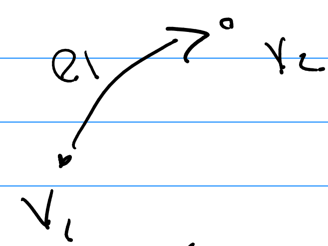
Graph theory

graph = set of vertices (non-empty)
set of edges

Visualize:

undirected graph | directed graph



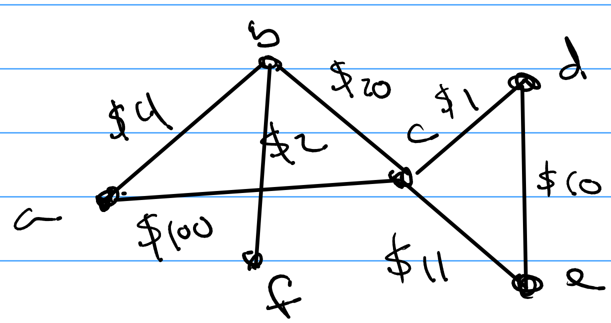


$e_1 = \{v_1, v_2\}$ unordered pair

$e_1 = (v_1, v_2)$ ordered pair

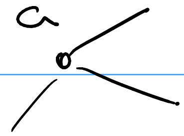
Weighted graph: graph (+) a $\boxed{\text{weight function}}$

from edges to weights



$w(\{a,b\}) = \$4$

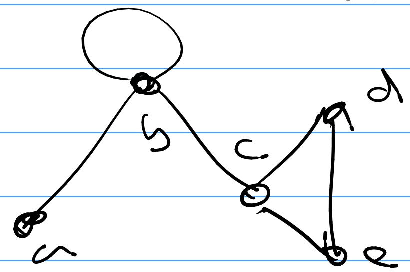
Properties.



$\deg(a) = 3$

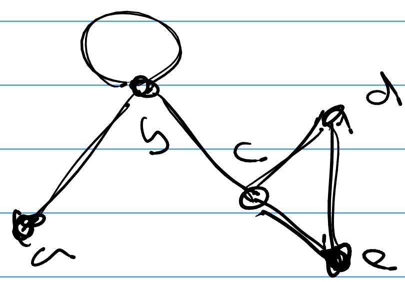
degree of $a = \deg(a)$

= # of edges attached to it, but loops count as 2.



$\deg(a) = 1$
$\deg(b) = 4$
$\deg(c) = 3$
$\deg(d) = 2$
$\deg(e) = 2$

Path: seq. of edges



a, b, b, c, d, e, c

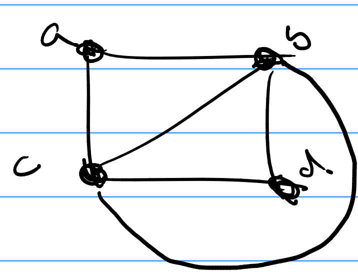
Special paths:

① circuit (begin/end at same vertex)

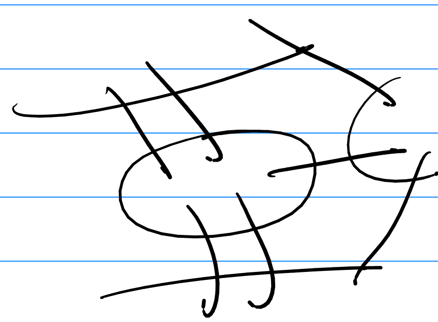
ex $\boxed{e, d, c, e}$

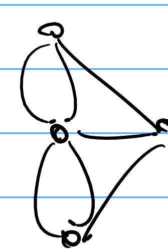
Simple = each edge in path is used once

② Euler path & Euler circuit



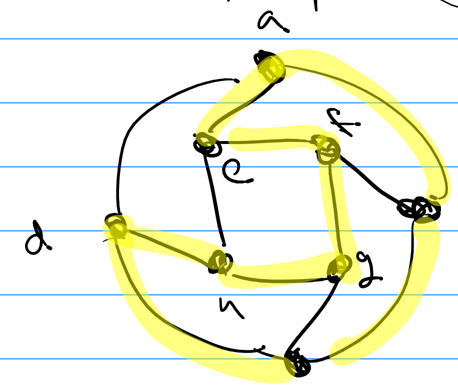
a, b, d, c, b, c, a





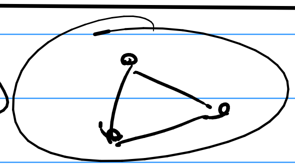
(3) Hamilton path / circuit

simple path / (circuit) that visits every vertex 'once'

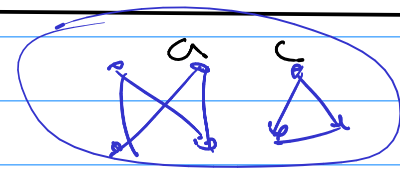


a, e, f, g, h, d, c, b, a

Connected



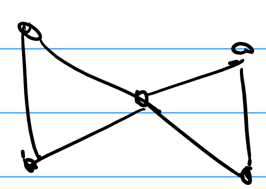
is connected

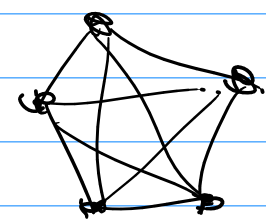


is not connected

if there is a path to/from every pair of vertices → connected.

strength of connectedness





K_5 (complete graph)

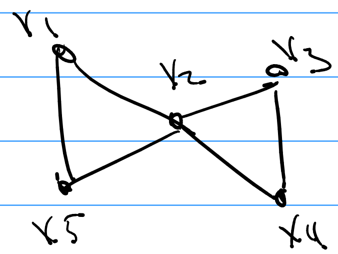
$\kappa(G)$ = vertex cut = minimal number of vertices to cut out that makes the graph disconnected

$\lambda(G)$ = edge cut = min number of edges to cut out that makes the graph disconnected

Counting paths

① Adjacency Matrix : $A = \begin{array}{c|c} & v_1 \; v_2 \cdots v_k \\ \hline \begin{matrix} v_1 \\ v_2 \\ \vdots \\ v_k \end{matrix} & \begin{bmatrix} & & \\ & a_{ij} & \\ & & \end{bmatrix} \end{array}$

a_{ij} = # of edges from v_i to v_j



$$A = \begin{bmatrix} 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \end{bmatrix}$$

$A^r = [r_{ij}]$ $\quad$ r_{ij} = # of paths from v_i to v_j

F, W, G, C, ϕ

FWGC | | ϕ

(FWGC, ϕ)

(WC, FG)

(FWGC, $\emptyset$) $\xrightarrow{\text{path}}$ ($\emptyset$, FWGC)

~~(WGC, F)~~

~~(WG, FC)~~