Final ! 16 probs → 4 per exam → 150 pts = 100%

MATH 321 ... FINAL EXAM REVIEW

EXAM 1 PROBLEMS

!O → on exam

X → not on exam

?O → Maybe? on exam

1) Construct the truth table everyone should know.

show all ops

2) a) Let c : "The cat scared the dog", s : "The cat is named Silly", and p : "Silly has a pet lion". Express the compound proposition $(s \to (c \wedge p))$ as an English sentence. Use the words "necessary" and/or "sufficient" where appropriate for the implications instead of using the words "if/then".

sym ⇄ eng

b) Express "For the mouse to defeat the cat it is sufficient yet not necessary that it drinks lots of coffee" using propositional symbols and logical operators.

3) Use a truth table to show that the statements $[(p \to q) \wedge (p \to r)]$ and $[p \to (q \wedge r)]$ are logically equivalent.

$\leftrightarrow$

we will have a prob with logical equiv. (use or show)

4) Show that $(p \to q)$ and $(\neg q \to \neg p)$ are logically equivalent by discussion.

5) Use logical equivalences to show that $(p \wedge q) \to p$ is a tautology.

$(p \wedge q) \to p \equiv \neg(p \wedge q) \vee p$

$\equiv$ etc...

6) Let $S(u)$ mean that "u is silly," $F(v)$ mean that "v is fast," and $B(a, b)$ mean that "a has beat b in a race", where the universe of discourse for each variable consists of all children.

a) Express $\exists x(S(x) \wedge \forall y(F(y) \to B(x, y)))$ by a simple English sentence.

b) Use quantifiers and the propositional functions given above to express "Every fast kid has either beat John in a race or been beat by John in a race".

?

7) Is the following argument valid? "You do not do every problem in the book or you learn discrete mathematics. You learned discrete mathematics. Therefore, you did every problem in the book." Explain your answer.

Valid?   invalid?

8) Come up with three valid conclusions for the set of premises: "If I drink coffee at bedtime, then I have strange dreams." "I have strange dreams if there is music playing while I sleep." "I did not have strange dreams." "Having strange dreams is sufficient for me to pass Math 321." Explain your answers.

1

9) Prove that $\sqrt{2}$ is irrational. Remember to prove any lemma you use.

Lemma: n^2 is even $\to$ n is even

10) For the integers 2,3,4,... Prove: if $n^2 < 2^n$, then $n > 4$.

11) Show that there exist irrational numbers x and y such that, x^y is rational.

Exam 2 Problems

1) Use set builder notation and roster forms to represent each of the following sets. The set A is even integers from 1 to 10, the set B is all integers that are a multiple of 4 from -10 to 10, and among a universe of discourse of integers from -10 to 10. And then illustrate all the sets and the universe of discourse with a single Venn Diagram.

?

2) For $A = \{a\}$ and $B = \{1, 2\}$ find $P(A \times B)$.

Power set?

Cross product?

3) Represent $A \cap \overline{(A \cap B)}$ with a Venn Diagram by using a membership table.

4) Show that $\overline{(A \cup B)} = \bar{A} \cap \bar{B}$ using set builder notation and logical equivalences.

ex $\overline{A \cup B} \cap C = \{e \mid (e \in \overline{A \cup B}) \wedge (e \in C)\}$

$= \{e \mid \neg(e \in A \vee e \in B) \wedge e \in C\} = \text{etc}$

5) If f and $f \circ g$ are onto, does it follow that g is onto? Justify your answer.

6) a) Find a function $f : ∷^+ \to ∷$ where f is one-to-one and not onto.

b) Find a function $f : ∷ \to ∷^+$ where f is not one-to-one and is onto.

7) Sequences ...

a) List the first 5 terms of the sequence $a_0 = -1, a_1 = 1$ and $a_n = a_{n-1} + 2a_{n-2}$.

b) Find formulae for the sequence: 3, 6, 12, 24, 48, ...

8) Find the value of the sum ...

$$\sum_{k=50}^{100} k = \boxed{\sum_{k=1}^{100} k - \sum_{k=1}^{49} k} = \text{etc}$$

$50 + 51 + \cdots + 100$

$100 + 99 \cdots + 50$

$150 + 150 + \cdots + 150 = 51(150)$

$\frac{51(150)}{2}$

6

9) Prove that [illegible] is countable.

one of these

10) Prove that [illegible] is uncountable.

11) Find $A+B$, $A \cdot B$, $A \vee B$, $A \wedge B$, and $A \odot B$ if ...

$$A = \begin{pmatrix} 0 & 1 \\ 0 & 1 \end{pmatrix} \quad B = \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix}$$

Exam 3 Problems

use

$\square \mid \Delta$

$\to \square \cdot k = \Delta$

$k \in \mathbb{Z}$

?

1) Given $a, b,$ and c are integers with $c \neq 0$, Show that if $ac|bc$, then $a|b$.

$ac|bc$

$\to ack = bc \quad k \in \mathbb{Z}$

$\to ak = b \to a|b$

?

2) a) Find $-17 \operatorname{div} 7$ and $-17 \bmod 7$

$a = d \cdot b + r$

$0 \leq r < b-1$

$d = a \operatorname{div} b \qquad r = a \bmod b$

remember

b) Find $17 \operatorname{div} 7$ and $17 \bmod 7$

c) List one negative integer and two positive integers that are congruent to $\boxed{-4}$ modulo 11.

d) Find $(7^{123}+8)^2 \bmod 6$

$(1^{123}+2)^2 \bmod 6 = \boxed{3}$

$7 \equiv_6 1 \qquad 8 \equiv_6 2$

plus, multiply

bases: 2, 7, $\boxed{11}$ $\to$ symbols $\{0, 1, 2, \ldots, 9, a\}$

!
0

3) Perform the requested operations ...

a) $(1,2,3)_7 + (4,5)_7$ using only base 7 numbers.

b) $(1,4)_7 \times (2,5)_7$ using only base 7 numbers.

4) Prove there are infinitely many primes.

2.0

5) Find the gcd and lcm of 140 and 75 using prime factorization.

6) Find the gcd of 140 and 75 using Euclid's Algorithm.

!0

7) Given the affine-shift function: $f(p) = (11p + 3) \bmod 6$ find the decryption function $f^{-1}(c)$.

11 under mod6

8) For a public key encryption $e = 11$ and $n = 119$. Find the decryption power d.

$\gcd(96, 11)$

$n = (p-1)(q-1)$

$= 6 \cdot 16 = 96$

$119 = pq = 7 \cdot 17$

9) Prove that $1/2 + 1/4 + 1/8 + \ldots + 1/2^n = 1 - 1/2^n$ for $n = 1, 2, 3, \ldots$ using weak induction.

1in

10) Prove all integers $n \geq 2$ can be written as a product of primes using strong induction.

!6

$\to$ have 1 weak induction proof.

11) Prove that $f_1 + f_3 + f_5 + \ldots + f_{2n-1} = f_{2n}$ when n is a positive integer.

Exam 4 Problems

?, 1) How many license plates can be made where a plate uses either three digits followed by four uppercase English letters or a plate uses two English letters (uppercase or lowercase) followed by five digits or a plate uses seven uppercase English letters? (Do not simplify your answer. Leave it as a product and/or sum of numbers.)

$$10^3 26^4 + 52^2 \cdot 10^5 + 26^7$$

?, 2) Given the integers from 13 to 94 (including 13 and 94) how many of them are divisible by 2? How many are divisible by 3? How many are divisible by 2 and 3? How many are divisible by 2 or 3?

div by 6

by 2 + by 3 − by 6

?, 3) Use the generalized pigeonhole principle to find the minimum number of students who have to come to class to be sure that at least five have the same grade in an A, B+, B, C+, C, D+, D, and F grading system.

$$\left\lceil \frac{N}{8} \right\rceil = 5 \qquad 4 \cdot 8 + 1 = \boxed{33}$$

4) How many distinct points (x, y) with integer coordinates are needed in the xy plane to have a midpoint joining at least one pair of these points with integer coordinates? (Explain)

See lectures.

5) (Please leave your answers in factorial notation) 9 people (5 guys and 4 girls) show up for a basketball game.

?, a) How many ways are there to choose 5 players to play if at least two players must be a girl?

b) How many ways are there to rather simply pick 5 players to play?

$$P(9,5) = \frac{9!}{4!}$$

6) (Please leave your answers in factorial notation) How many bit strings of length ten ...

a) ... Have more 0's than 1's?

b) ... Have at least seven 1's?

7) What is the 32nd term for $(x^2 + x^{-2})^{42}$? Leave your coefficient in factorial notation, but combine the variables together to get a single x to a specific power.

8) Prove $\binom{n+1}{k} = \binom{n}{k} + \binom{n}{k-1}$ by any ~~method~~. combinatorial proof

9) Find a recurrence relation with initial conditions for the number of ways to lay out a walkway with slate tiles if the tiles are red, green, or black, so ~~that no two green~~ tiles are adjacent and tiles of the same color are considered indistinguishable.

tile? bitstrings? steps? money?

10) Solve $a_n = a_{n-1} + 2a_{n-2}$ with initial conditions $a_0 = 4$ and $a_1 = -1$.

~~11) Solve $a_n = 8a_{n-2} - 16a_{n-4}$.~~