Math 322

Set of all functions $f: \mathbb{R} \to \mathbb{R}$

$R = \{ (a,b) \mid a' = b' \text{ for all values in their domain} \}$

$\boxed{\text{ex}}$ $\frac{d}{dx}[x^2+1] = 2x$

So $(x^2+1) R (x^2-4)$

$\frac{d}{dx}[x^2-4] = 2x$

$\boxed{\text{Equiv Relation?}}$

1. Reflexive $\forall a (aRa) \equiv (a' = a')$ true

2. Sym $\forall a \forall b (aRb \to bRa)$

$\equiv (a' = b' \to b' = a')$ true

3. trans $\forall a \forall b \forall c (aRb \wedge bRc \to aRc)$

$\equiv (a' = b' \wedge b' = c' \to a' = c')$

$\boxed{\text{true}}$

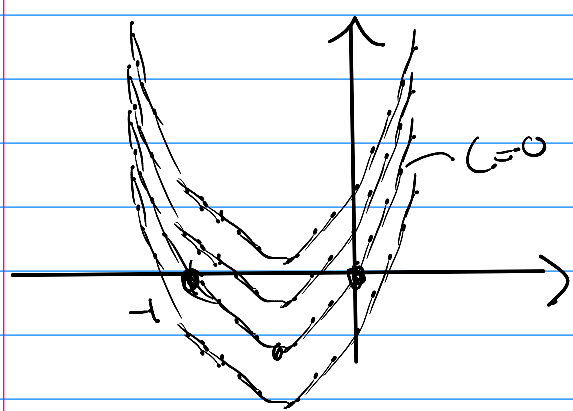
R is an equiv. relation.

So $[x^2+x]_R = \boxed{?}$

$(x^2+x)' = 2x+1 = \{\ ?\ \}'$ all functions whose derivative is $2x+1$

$$[x^2+x]_R = \{ f \mid f = x^2+x+c,\ c \in \mathbb{R} \}$$

$$\int (2x+1)\,dx = x^2+x+c$$



c=0 $\quad x^2+x$

$\boxed{9.6}$ order : reflexive, antisymmetric, transitive

Def: if R on set S is ref, antisym, and transitive we call it a partial ordering.

① S with its partial ordering is a poset : (S, R)

② S is called a partially ordered set.

Notation: R is a partial order use $\preccurlyeq$

$\boxed{\text{Poset } (S, \preceq)}$

$(a,b) \in R$ write $a \preceq b$
$(b,a) \in R$ write $b \preceq a$

a, b are <u>comparable</u>

if $a \npreceq b$ and $b \npreceq a$ a, b are <u>incomparable</u>

ex $(\mathbb{Z}, \leq)$

<u>ref:</u> $\forall a R a \equiv \forall a (a \leq a)$ <u>true</u>

<u>antisym</u> $\forall a \forall b (aRb \land bRa \rightarrow a = b)$

$\equiv \forall a \forall b (a \leq b \land b \leq a \rightarrow a = b)$ <u>true</u>

<u>trans:</u> (you do @ home) <u>true</u>

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why use partial?

$(\mathbb{Z}^+, |)$

$R = \{(x,y) \mid x \mid y\}$ divides

such that

$3 \mid 6$ true

$3 \mid 7$ false

refl: $\forall a (aRa) \equiv \forall a (a \mid a)$ true

antisym: $\forall a \forall b (aRb \wedge bRa \to a = b)$

$$\equiv ((a \mid b) \wedge b \mid a \to a = b)$$

$$\boxed{a \cdot c_1 = b} \wedge b \cdot c_2 = a \qquad (c_1, c_2 \text{ are ints})$$

$$\to a \cdot c_1 \cdot c_2 = a$$

$$\to c_1 \cdot c_2 = 1 \to 1 = c_1 = c_2$$

so $a = b$ $\boxed{\text{true}}$

trans: $aRb \wedge bRc \to aRc$

$$\equiv a \mid b \wedge b \mid c \to a \mid c$$

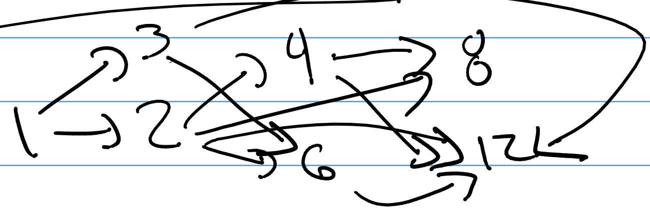
$$\equiv (a \cdot c_1 = b \wedge b \cdot c_2 = c) \to a \mid c$$

$$a \cdot \boxed{c_1 \cdot c_2} = c$$

$\boxed{\text{true}}$

$(\mathbb{Z}, \leq)$ $\boxed{1 \to 2 \to 3 \to 4}$

$(\mathbb{Z}^+, \mid)$



partial !

(get rid of incomparable)

Def: Total Order is a partial order and every two elements of S are comparable

$(S, \preceq)$ is a total order

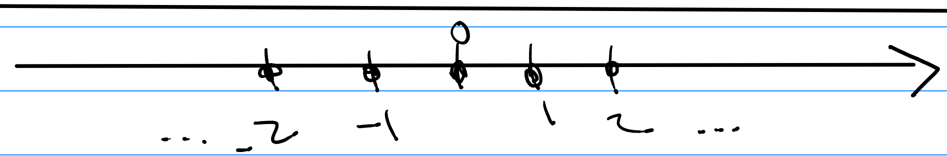
S is totally ordered (linearly ordered)

$\preceq$ is a total order (linear order)

also called a $\boxed{\text{chain}}$

So $(\mathbb{Z}, \leq)$ is a chain

$\boxed{\text{ex}}$ $(\mathbb{Z}^+, |)$ is not a chain



$(\mathbb{Z}, \leq)$

$\boxed{Q}$ what is the 1^{st} $\mathbb{Z}$?

$\boxed{\text{Def}}$ $(S, \preceq)$ is well-ordered if it is a total order and every subset of S has a least element.

$\boxed{\text{ex}}$ $(\mathbb{Z}, \leq)$ is not well ordered

$(\mathbb{Z}^+, \leq)$ is well ordered.

Reminder: (Induction)

$$[\boxed{P(1^{st})}^{T} \wedge \boxed{\forall k\, (P(k^{th}) \to P(k+1^{st}))}^{T}] \to \boxed{\forall n\, P(n)}^{F}$$

is a tautology. $\to$ why?

$\boxed{\text{th}^{m}}$ the principle of well-ordered induction.