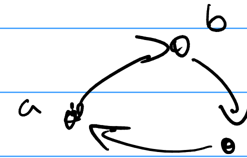
# Math 322

Q5/ 10.4 #11 directed graph

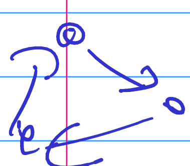


A) Strongly connected : for u,v path to and from each distinct pair.

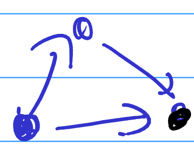
B) weakly connected : ignore direction
( consider the underlying undirected graph)
if its "undirected" version is connected
→ call G weakly connected

weakly connected

Strong and weak

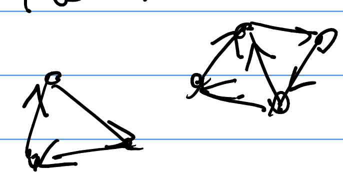


¬strong is weak

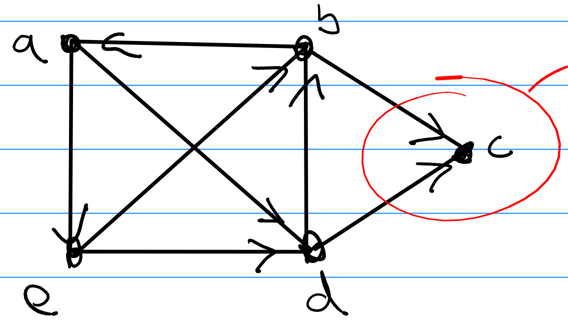


~~Strong ¬weak~~

¬strong ¬weak



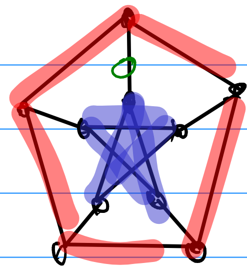
11b)

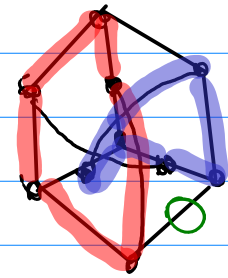


¬ Strong

is weakly connected

10.3 #13





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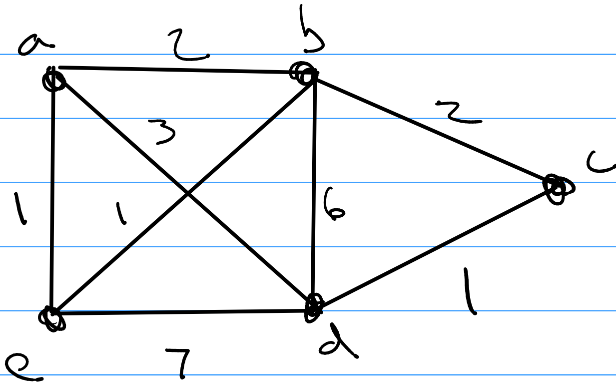
## 10.6 Shortest Paths.

Weighted Graphs $G = (V, E)$

add a weight function $w : E \to \mathbb{R}$

$w(e_i) = \#$ (weight / cost)

ex





$w(\{a,b\}) = 2$

$w(\{a,c\}) = 1$

etc

before: path length = # of edges

Now: path length = sum of weights

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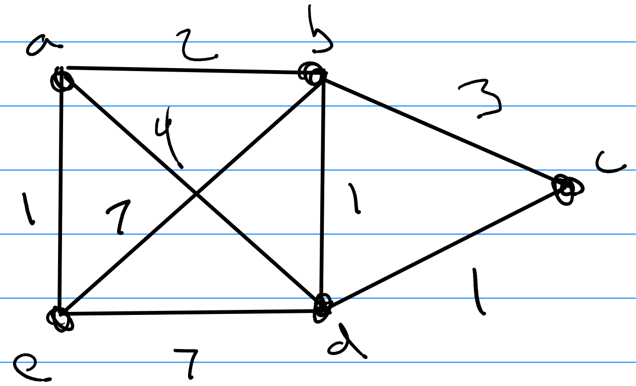
Problem: Find paths of least cost

(shortest path problem)

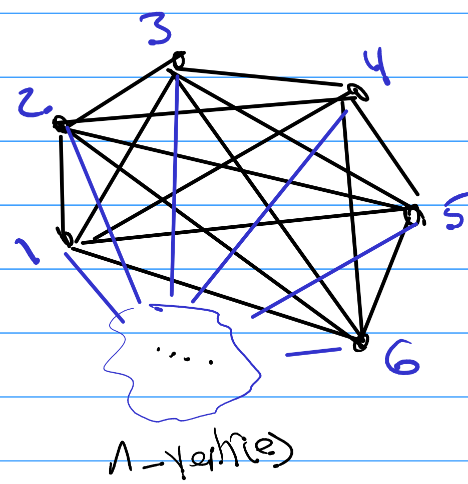
① given vertex $v_1$ → find all shortest paths starting @ $v_1$ to all the other vertices.

ex 



shortest paths ⤵
c to a = ?
c to b = ?
c to c = ?
c to d = ?
c to e = ?

② Traveling Salesman Problem

$K_n$ 



$\deg(v) = n-1$

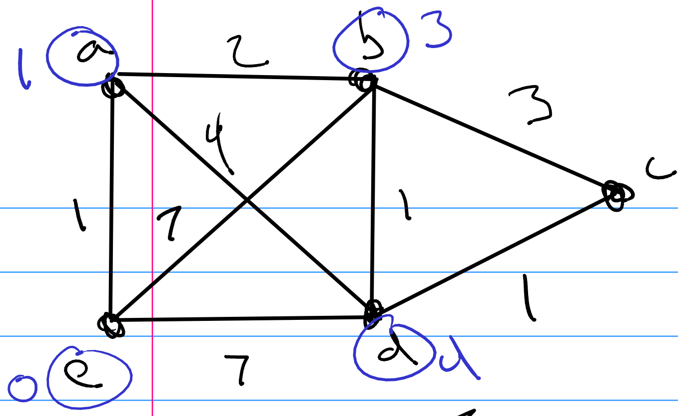
→ Ore's / Dirac's

Have a Hamilton Circuit.

$|\text{total Hamilton circuits}| = (n-1)(n-2)\cdots(1)$
$= (n-1)!$

→ weighted $K_n$, for all $(n-1)!$ Hamilton circuits pick the least cost one.

by symmetry you check $\dfrac{(n-1)!}{2}$




shortest paths from e

Dijkstra's Algorithm

| e | path | cost |
|---|---|---|
| e | e | 0 |
| a | e,a | 1 |
| b | e,a,b | 3 |
| d | e,a,b,d | 4 |
| c | e,a,b,d,c | 5 |

found to ?

b, c : 6

d, c : 5 ✓

Cost $O(n^2)$